B.M.S COLLEGE FOR WOMEN AUTONOMOUS
BENGALURU – 560004

END SEMESTER EXAMINATION – OCTOBER 2022

M.Sc. in Mathematics – II Semester
Numerical Analysis I

Course Code: MM205T   **QP Code: 21005**
Duration: 3 Hours   **Max marks: 70**

Instructions: 1) All questions carry equal marks.
                    2) Answer any five full questions.

1. a) Show that the fixed point iteration method has a linear rate of convergence. Hence, find the real root of $x^3 + 4x^2 - 10 = 0$.
    b) Find the smallest root of the equation $x^3 - 9x^2 + 26x - 24 = 0$ using Ramanujan's method.

                                                                (7+7)

2. a) Perform two iterations of the Bairstow's method to estimate the quadratic factor $x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$ with $p_o = q_o = 2$.
    b) State Descarte's rule of signs and Sturm's theorem . Find the number of real and complex roots of $x^3 - 5x + 1 = 0$.

                                                                (7+7)

3. a) Using Crout's method, solve the linear system of equations:
       $3x + y + z = 5; \ x + 3y + z = 5; \ x + y + 3z = 5.$
    b) Explain the terms "ill-conditioned" and "well-conditioned" with suitable examples. Show that the Hilbert matrix of order 3 is highly ill-conditioned by finding its condition number.

                                                                (7+7)

4. a) Solve the linear system of equations by using Thomas algorithm:
       $x_1 + 2x_2 = 5; \ 2x_1 + 3x_2 - x_3 = 5; \ 4x_2 + 2x_3 + 3x_4 = 26; \ 2x_3 - 4x_4 = -10.$
    b) Find the roots of the following system by Newton Raphson method
    $x^2 - y^2 = 4; x^2 + y^2 = 16$ with given initial conditions $x_o = y_o = 2\sqrt{2}$
    (Perform two   iterations).

                                                                (7+7)

5. a) Obtain the Lagrange's interpolating polynomial of degree n in its standard form.
    b) Find the rational approximation $R_{4,5}$ for the function $f(x) = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9}$.
     Compare $f(0.4)$ with the value obtained using the rational approximation.

                                                                (7+7)

6. a) Obtain the Newton's bivariate interpolation polynomial for the following table and hence find $f(0.5, 0.5)$.

y/x	x_0	x_1	x_2
y_o	1	3	7
y_1	3	6	7

y_2	7	11	17

b) Obtain the natural cubic spline approximation for the following data:

x	1	2	3
$y = f(x)$	-9	-2	17

Compute $f(1.5)$ and $f(2.5)$.

(7+7)

7. a) Derive Gauss-Hermite two and three-point quadrature formula.
   b) Evaluate:
   (i) $\int_0^1 \frac{dx}{1+x}$ using Gauss-Legendre two and three point formula.
   (ii) $\int_{-1}^1 (1 - x^2)^{\frac{3}{2}} \cos x \, dx$ using Gauss-Chebyshev two and three point formula.

(7+7)

8. a) Derive Gauss-Lagaurre two and three-point quadrature formula.

   b) Evaluate $\int_1^5 \int_1^5 \frac{dx\,dy}{\sqrt{x^2+y^2}}$ using trapezoidal and Simpon's rule with two sub intervals.

(7+7)

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